Characterizing the Core of Mercury

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The preferred source of Mercury's magnetic field (Ness, *et al.*, 1974) is a dynamo (Ness, *et al.*, 1975), which in turn implies that Mercury has a currently molten core. However, whether or not a given model with differentiation has a core with at least a conducting, molten outer layer that persists to the present depends critically on assumptions about the time of core formation, the time varying distribution of radioactive elements within the planet, core composition, thermal conductivities and the efficiency of convective heat transport. (Siegfried and Solomon, 1974; Fricker *et al.*, 1976; Cassen *et al.*,1976; Schubert *et al.* 1988), The reservations about the persistence of a molten core and the profound implications for thermal histories of terrestrial planets, the theory of planetary dynamos, and rotational history of Mercury makes the existence and size of the suspected fluid core probably the most important information that could be obtained about the planet Mercury. We review past assertions below that the determinations of the four parameters, $C_{20},\ C_{22},\ \theta,\ \phi$, are sufficient to determine the size and state of Mercury's core (Peale, 1976, 1981, 1988). C_{20} and C_{22} are gravitational harmonics, θ is Mercury's obliquity and ϕ is the amplitude of the forced, 88 day period libration in longitude. The existence of gravity assisted spacecraft trajectories to Mercury with modest propulsion systems (*e.g.*, Yen, 1989), and the possibility of determining these parameters with instrumentation on a Mercury-orbiting spacecraft (Wu, *et al*, 1996; Zuber and Smith, 1997) makes a re-examination of this proposal particularly relevant.

There are two necessary conditions on the core–mantle interaction for the experiment to work: 1. The core must *not* follow the 88 day physical librations of the mantle. 2. The core must follow the mantle on the time scale of the 250,000 year precession of the spin in Cassini state 1 (See Peale, 1969 for discussion of Cassini states). We shall assume these two conditions are satisfied to develop the method and later establish the constraints on the core viscosity for which they are satisfied.

The physical libration of the mantle about the mean resonant angular velocity arises from the periodically reversing torque on the permanent deformation as Mercury rotates relative to the Sun. The amplitude of this libration is given by (Peale, 1972),

$$\phi = \frac{3}{2}\left(\frac{B-A}{C_m}\right)\left(1 - 11e^2 + \frac{959}{48}e^4 + \cdots\right), \quad (1)$$

where the moment of inertia in the denominator is that of the mantle alone since the core does not follow the librations. The core is assumed axially symmetric so it does not contribute to $B-A$. Dissipative processes will carry Mercury to rotational Cassini state 1 (where spin, orbit precessional and orbital angular velocities remain coplanar) with an obliquity θ close to 0° (Peale, 1988). This leads to a constraint,

$$K_1(\theta)\left(\frac{C-A}{C}\right) + K_2(\theta)\left(\frac{B-A}{C}\right) = K_3(\theta) \quad (2)$$

where the moment of inertia in the denominator is now that of the total planet since the core is assumed to follow the precession. Note that the precession here is not the relatively rapid precession of the spin about the Cassini state in the frame rotating with the orbit, which the core is not likely to follow, but it is the precession of the orbit (with the much longer period) in which frame the spin is locked if Mercury occupies the exact Cassini state.

The lowest order gravitational harmonics are expressed in terms of the moment differences as follows.

$$C_{20} = -\frac{C-A}{M_p a_e^2} + \frac{1}{2}\frac{B-A}{M_p a_e^2} == -(6.0 \pm 2.0) \times 10^{-5},$$
$$C_{22} = \frac{B-A}{4M_p a_e^2} = (1.0 \pm 0.5) \times 10^{-5}, \quad (3)$$

where the numerical values are estimated for Mercury from Mariner 10 flyby data (Anderson, *et al*, 1986). Eqs. (3) can be solved for $(C-A)/M_p a_e^2$ and $(B-A)/M_p a_e^2$ in terms of C_{20} and C_{22}, determined by tracking the orbiting spacecraft. Substitution of the solutions of Eqs. (3) into Eq. (2) yields a numerical value for $C/M_p a_e^2$ since the K_i are known once the obliquity θ is measured.

Measurement of the amplitude ϕ of the physical libration determines $(B-A)/C_m$ (Eq. (1)) from which three known factors give

$$\left(\frac{C_m}{B-A}\right)\left(\frac{B-A}{M_p a_e^2}\right)\left(\frac{M_p a_e^2}{C}\right) = \frac{C_m}{C} \le 1. \quad (4)$$

A value of C_m/C of 1 would indicate a core firmly coupled to the mantle and most likely solid. If the entire core or the outer part is fluid, $C_m/C \simeq 0.5$ for the large core size ($r_c \simeq 0.75R_☿$) in current models of the interior (Cassen *et al.*, 1976).

Are the necessary condition on the core–mantle interaction likely to be satisfied? Two time constants for the decay of a differential rotation of a spherical cavity and its contained fluid are used to relate the coupling constant between the core and mantle to the kinematic viscosity (Peale, 1988).

$$\tau = \frac{r_c}{(\nu\dot{\psi})^{1/2}} \qquad \tau = \frac{r_c^2}{\nu}, \tag{5}$$

where the first applies to small viscosities and the latter to large viscosities with $\dot{\psi}$ being Mercury's spin rate. If $\tau \gg$ 88 days, the core will not follow the mantle, and if $\tau \ll 250,000$ years, the core will follow the precession of the mantle angular momentum. These conditions correspond to

$$4 \times 10^{-4} < \nu < 5 \times 10^8 \text{ to } 4 \times 10^9 \text{ cm}^2/\text{sec}. \tag{6}$$

As this range includes all possible values for the viscosity of likely core material (*e.g.* Gans, 1972), the experiment should work if the core–mantle coupling is primarily of a viscous nature.

Measured ranges of values of C_{20}, C_{22} are given in Eqs. (3), and values of the obliquity and of the libration amplitude corresponding to the extremes values of these harmonics are

$$1.7 \lesssim_m \theta \lesssim_m 2.6 \text{ arcmin} \quad 20 \lesssim_m \phi \lesssim_m 60 \text{ arcsec}, \tag{7}$$

where θ follows from the solution of Eq. (2) and ϕ from Eq. (1) with $\phi = .854(B-A)/C_m$ for $e = 0.206$ and $C_m/C = 0.5$ and $C/M_p a_e^2 = 0.35$ being assumed.

To estimate the required precision of the measurements for meaningful interpretation, we designate the four parameters whose nominal values are given in Eqs. (3) and (7)) by η_i and write

$$\Delta\left(\frac{C_m}{C}\right) = \sum_i \frac{\partial}{\partial\eta_i}\left(\frac{C_m}{C}\right)\Delta\eta_i, \tag{8}$$

which gives a maximum uncertainty of

$$\frac{\Delta(C_m/C)}{(C_m/C)^0}|_{max} = \sum_i \left| f_i \frac{\Delta\eta_i}{\eta_i^0} \right| \approx 22\%, \tag{9}$$

where the superscript 0 indicates the nominal values of the parameters, $f_i = -0.83, 0.83, -1, -1$ respectively for the terms in Eq. (9) with $\eta_i = C_{20}, C_{22}, \theta, \phi$, and where the numerical value corresponds to fractional uncertainties of 0.01, 0.01, 0.1, 0.1 respectively for the four parameters with nominal values $C_{20}^0 = -6 \times 10^{-5}$, $C_{22}^0 = 1 \times 10^{-5}$, $\theta^0 = 3$ arcmin, and $\phi_0 = 30$ arcsec being assumed. If instead of the second of Eqs. (3), we assume $C_{22} = -C_{20}/8$ and $C/M_p a_e^2 = 0.35$, then $5.2 \gtrsim \theta^0 \gtrsim 1.0$ arcmin for $2 \times 10^{-5} \lesssim_m -C_{20} \lesssim_m 1 \times 10^{-4}$ with the corresponding range of ϕ^0 being shifted only slightly to smaller values. But because the angles are all small, the f_i are not significantly affected in the uncertainty estimate provided C_{22}/C_{20} remains approximately constant.

The maximum error would yield $C_m/C = 0.5 \pm 0.11$ which would distinguish the molten core. Since the numerical coefficients in Eq. (9) are not very sensitive to the nominal values of the parameters, the error estimates remain the same for other reasonable values of the parameters if the *fractional* uncertainty of each parameter is unchanged. If, on the other hand, we fix each parameter uncertainty to be that value derived from the above assumed fraction of the central value but let the nominal values go to the extremes in Eq. (9), $C_m/C = 0.5 \pm 0.24$ for all minimal nominal values, and $C_m/C = 0.5 \pm 0.065$ for all maximal nominal values. Even the worst case would distinguish a molten core, although the core size would not be well constrained.

Measurement of C_{20} and C_{22} to two significant figures and $\Delta\theta$ and $\Delta\phi$ to a few arcseconds will assure that meaningful bounds on C_m/C are obtained.

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